# Weather Derivatives[#]

## *Jan Pígl*[*]

## Introduction

This work considers weather derivatives and their pricing. The aim of this work is the definition of weather derivatives and the way how to price them. We show as well that linear and nonlinear models of time series of temperatures measured in Prague and in Brno have not good results in the estimation of parameters $\mu_I$ and $\sigma_I$ of the probability distribution function *P(I)* of the weather index which is essential in their pricing.

## Definition of weather derivatives

Weather derivatives are a relatively new concept. The first contract was made in 1997. The total volume was estimated as 500 million US dollars. The first stock exchange for weather derivatives is the Chicago Mercantile Exchange which established weather indices based on the weather in 11 different cities in the USA. The big players are energy companies and insurance companies. In short weather derivatives are a form of insurance, or in financial terms an instrument which a company can use to hedge its exposure to fluctuations in weather.

The idea of weather derivatives may be understood through a simple example. A energy company selling electricity for heating may want to protect themselves against losses due to unexpected moderate conditions the following winter. It can then buy an insurance policy, weather derivative, from a financial institution which agrees to pay out a certain amount of money if winter should turn out to be moderate. Note that energy company reduces its losses in adverse conditions if it buys weather derivative, but also it reduces its gains by cost of the insurance premium if the winter turns out to be cruel. The aim of weather derivatives is to straighten out the temporal fluctuations in the company's revenues, whose revenues are dependent on weather.

Two components are necessary in order to put up a weather derivative contract a weather index and a payout function. The weather index, denoted *I*, characterises the weather over a certain period, called the *strike* period. Weather derivatives are usually based on temperature. In practice weahter derivatives are not based on raw temperature but a more convenient way is to use heating or cooling degree days (HDDs or CDDs), defined as:

$$HDD_i = \max(T^* - T_i, 0) , \qquad\qquad\qquad\qquad\qquad (1)$$

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$$CDD_i = \max(T_i - T^*, 0) ,\tag{2}$$

where   $i$   –   indicates a specific day,
  $T_i$   –   the average temperature measured on that day,
  $T^*$   –   fixed reference temperature (18°C in Europe).

In the example above, the weather index might be defined as the total number of HDDs over the winter season:

$$I = \sum_{i=i_f}^{i_f + N - 1} HDD_i ,\tag{3}$$

where   $i_f$   –   the first day of the season (strike period),
  $N$   –   the length of the season.

The payout function *Q(I)* determines how much financial (insurance) institution pays out for a given weather index result. If we go back to our example, we will see that we won´t expect payout if the winter is cruel (high *I*) and we will expect payout if the winter is moderate (low *I*). A typical payout structure has the form shown in Fig. 1, which displays a zero-payout threshold and a linear increase in payout below the threshold. The position of the threshold and the slope of the linear part must be agreed on by the two parties doing business. The payout function is overlaid a hypothetical Gaussian index probability distribution function (the probability density function *P(I)* is not drawn in the scale in the Fig. 1).

**Figure 1: The payout function Q(I)**
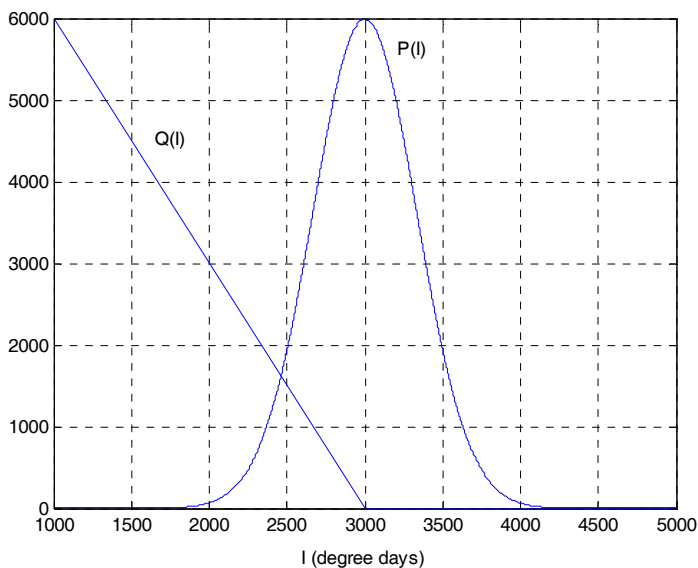


Source: Own calculations.

We have seen that the actual payout from the contract depends on the final value of the weather index. The value of the weather index itself depends on the weather. Now we have to solve the problem in which we have to decide how to price the weather

derivative because we have to know how much financial institution should charge the company. The usual way how to price the weather derivative (see Caballero, 2003, Caballero – Jewson – Brix, 2002) is to set:

$$S = E(Q) + R,$$ (4)

where   $S$   –   the price of the weather derivative at the time moment when we agree the contract (weather derivative).

The first term is the mean pay out:

$$E(Q) = \int_0^\infty Q(I)P(I)dI,$$ (5)

where   $P(I)$   –   the probability distribution function of the weather index.

The second term ($R$) is a risk premium. This term compensates the risk which financial institution takes in selling the weather derivative.

## Factors influencing the price of the derivative

The main features of $P(I)$ are captured by its first two moments, the mean $\mu_I$ and the variance $\sigma_I^2$. From Fig. 1 we can see that, if either the mean is overestimated or the variance underestimated, the derivative will be underpriced. In this example financial corporation loses money.

Let us then consider what aspect of daily temperature variability influences the values of $\mu_I$ and $\sigma_I^2$ (see Caballero – Jewson – Brix, 2002). We assume that the temperature over the winter period is always below $T^*$. We can then write:

$$HDD_i = T^* - T_i = T^* - E(T_i) - T_i',$$ (6)

where   $E(.)$   –   indicates an expected value,

and we define the temperature anomalies $T_i' = T_i - E(T_i)$. Then:

$$\mu_I = E(I) = NT^* - \sum_{i=i_f}^{i_f+N-1} E(T_i),$$ (7)

Thus, to obtain an accurate estimate of the mean index value $\mu_I$ we need only an accurate estimate of the seasonal cycle $E(T_i)$.

The situation is more complicated for the variance. We have:

$$\sigma_I^2 = E\left[(I - \mu_I)^2\right] = E\left[\left(\sum_{i=i_f}^{i_f+N-1} T_i'\right)^2\right] = \sigma_T^2 \sum_{i,j=i_f}^{i_f+N-1} \rho_{i,j}$$ (8)

where   $\sigma_T^2$   –   variance,

$\rho_{i,j}$   –   autocorrelation function of temperature anomalies.

Assuming the process is stationary, the autocorrelation function will only depend on the lag *k*. We than have:

$$\sigma_I^2 = \sigma_T^2\left(N + 2\sum_{k=1}^{N}(N-k)\rho_k\right) \tag{9}$$

where  $\sigma_T^2$  –  variance,

$\rho_{i,j}$  –  autocorrelation function of temperature anomalies,

Thus, to estimate the variance of *I* accurately, we need to estimate not only the variance of temperature anomalies, but also their autocorrelation structure up to lag *N*. If the autocorrelation structure is underestimated, the index variance will be too. The minimum requirements for a suitable time-series model are that it should correctly capture the seasonal cycle, the anomaly variance, and the anomaly autocorrelation structure out to lags of a season.

# Comparing the performance of linear and nonlinear models[1]

In this part we try to find the convenient models of time series of raw near-surface temperatures which capture the minimum requirements shown above. We approach the problem by making the null hypotesis that the model is perfect (that is, that the historical data are actually generated by the model itself) and then try to reject the hypotesis.

We will work with two data sets. The first data set is the set of the raw daily near-surface temperatures measured as the average temperatures for all day from the first of July 1993 up to the thirty-first of January 2007 in Prague. The second data set is the set of the raw daily near-surface temperatures measured at 2.00 PM from the first of July 1993 up to the thirty-first of January 2007 in Brno.

At first we will start with linear models. In Fig. 2 we see the selective autocorrelation function (ACF) of time serie of temperatures measured in Prague and time series of temperatures measured in Prague, we see that selective ACF of time serie has the statistical significant values for more then 110 lags which means that we can model this time serie with the ARFIMA model (fractionally integrated ARMA process). As the convenient model we will reason about the model ARFIMA(0,d,1). For estimated parameters of the model ARFIMA(0,d,1) see table 1. All parameters of the model are statistically significant on the basis of t-test but the model does not pass the Portmanteau test, the Asymptotic test and the ARCH test. The Portmanteau test says that the residuals of the model indicate autocorrelation. There is a possibility to cut off this autocorrelation in terms of linear model but we are not able to cut off conditional heteroskedasticity of residuals (which we tested by ARCH test) and unnormal distributoin of the residuals of the model (which we tested by Asymptotic test). Whence it follows that the linear models are not convenient to model the time series of temperatures so we have to try to find more convenient model of time series in the class of the nonlinear models.

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[1]  To get more information on the time  series models and the tests which we use in this work see Arlt, Arltová (2007).

We get very similar results for the time serie of the temperatures measured in Brno. In Fig. 3 we see selective ACF of time serie of temperatures measured in Brno, we see that selective ACF has the statistically significant values for more then 110 lags so we can model this time serie with ARFIMA. Again we will reason about the model ARFIMA(0,d,1). For estimated parameters of the model ARFIMA(0,d,1) see table 1. All parameters of the model are statistically significant on the basis of the t-test but the model does not pass the Portmanteuau test, the Asymptotic test and the ARCH test. It is the same situation as described in the previous paragraph. Whence it follows that we have to try to find more convenient model of time serie of temperatures in the class of the nonlinear models.

**Figure 2: ACF and TS of temperatures measured at Prague**
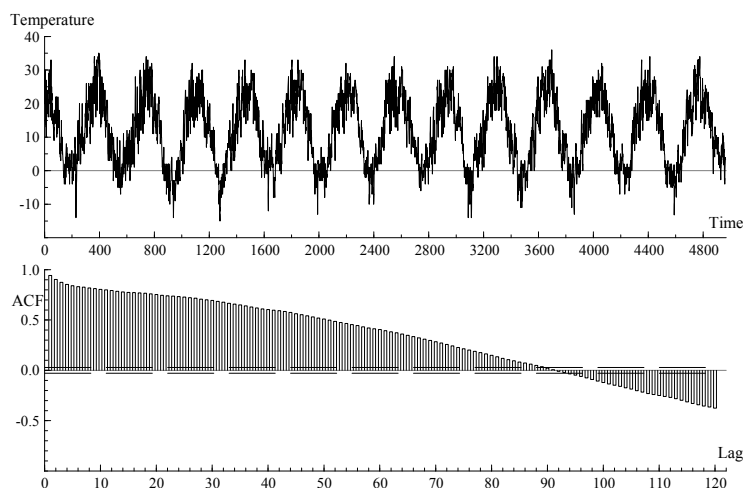


Source: Own calculations

**Figure 3: ACF and TS of temperatures measured at Brno**
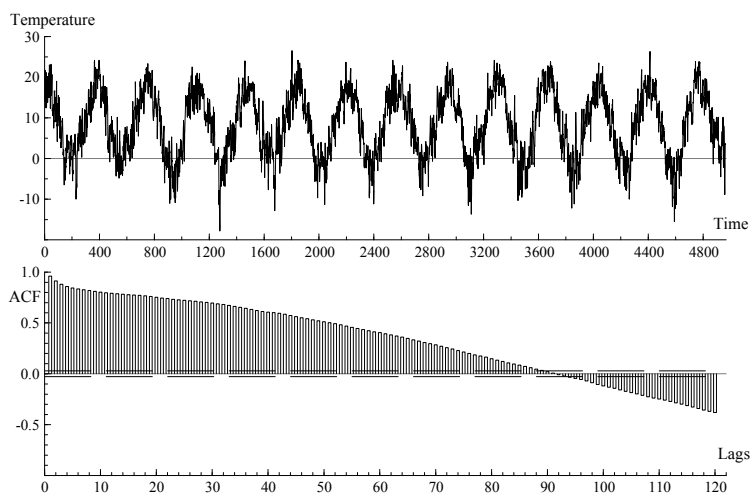


Source: Own calculations

**Table 1: Parameters of the ARFIMA model**

|  | ARFIMA(0,d,1) | |
|---|---|---|
|  | **d** | **MA(1)** |
| **Prague** | 0.678296 | 0.094182 |
| **Brno** | 0.683886 | 0.385876 |

Source: Own calculations

We have choosen two nonlinear models. The first one is the Self-Exciting Threshold Autoregressive model (SETAR) and the second one is the Markov-Switching model (MSW). The main difference is between these two models that we do not define the threshold variable in the Markov switching model. We define the threshold variable for the both time series of temperatures in the form:

$$v_{t,31} = \tfrac{1}{31} \sum_{i=0}^{31-1} X_{t-i} \; . \tag{10}$$

We can consider this characteristic as the measure of the volatility. The change of the autocorrelation structure of the stochastic processes can relate to their changeable variability and for this reason it is convenient to think of the threshold variable as we define it above. In this case this form is the moving average of the $X_t$ values and the value 31 is the length of month. The diagnostic control of SETAR and MSW models exploit standardized residuals.

In the first example we will consider the time serie of the temperatures measured in Prague. In the estimate of the parameters of the SETAR model we will think over two lags and two probability regimes. The test of nonlinearity shows the possibility of the use of the SETAR model. The estimated parameters of the model are in the table 2. All parameters of the model are statistically significant on the basis of t-test but the model does not pass the Portmanteau test, the Asymptotic test and the ARCH test analogously to linear model of the same time series. The probability regimes are separated by the threshold value 16,742 of the variable $v_{t,31}$. The difference in the estimate of the constants of the SETAR model shows that the mean value of the process is variable in time.

In the estimate of the parameters of the MSW model we will think over two lags and two probability regimes analogously to the SETAR model. The test of nonlinearity shows the possibility of the use of the MSW model. The estimated parameters of the model are in the table 2. All parameters of the model are statistically significant on the basis of t-test but the model does not pass the Portmanteau test and as well Asymptotic test. However, it passes the ARCH test. On this account this model is the most convenient from the all models of the class of linear and nonlinear models. In Fig. 4 we see the estimated time series of temperatures measured at Prague which we estimated by the MSW model and as well the probabilities of the regime one and the probabilities of the regime two whose processes diversify evidently in the period with the higher variability from the period with the lower variability.

In the second example we will consider the time series of the temperatures measured in Brno. In the estimate of the parameters of the SETAR model we will think over one lag and two probability regimes. The test of nonlinearity shows the possibility

of the use of the SETAR model. The estimated parameters of the model are in the table 2. All parameters of the model are statistically significant on the basis of t-test but the model does not pass the Portomanteau test, the Asymptotic test and the ARCH test analogously to the example above. The probability regimes are separeted by the treshold value 6.4462 of the variable $v_{t,31}$. The difference shows in the estimate of the constant of the SETAR model that the mean value of the process is variable in time.

In the estimate of the parameters of the MSW model we will think over one lag and two probability regimes. The test of nonlinearity shows the possibility of the use of the MSW model. The estimated parameters of the model are in the table 2. All parameters of the model are statistically significant on the basis of the t-test but the model does not pass the Portmanteau test, the Asymptotic test and the ARCH test analogously to SETAR model. In Fig. 5 we see the estimated time serie of temperatures measured at Brno which we estimated by MSW model and as well the probabilities of the regime one and the probabilities of the regime two.

**Table 2: Parameters of SETAR and MSW models**

| | SETAR | | | MSW | | |
|---|---|---|---|---|---|---|
| **Prague** | $\phi_{0.1}$ | $\phi_{1.1}$ | $\phi_{2.1}$ | $\phi_{0.1}$ | $\phi_{1.1}$ | $\phi_{2.1}$ |
| | 0.483094 | 0.845788 | 0.067971 | 0.1671 | 0.8644 | 0.0951 |
| | $\phi_{0.2}$ | $\phi_{1.2}$ | $\phi_{2.2}$ | $\phi_{0.2}$ | $\phi_{1.2}$ | $\phi_{2.2}$ |
| | 6.201851 | 1.358027 | 0.060079 | 1.8914 | 0.774 | 0.1176 |
| | SETAR | | | MSW | | |
| **Brno** | $\phi_{0.1}$ | | $\phi_{1.1}$ | $\phi_{0.1}$ | | $\phi_{1.1}$ |
| | 0.676376 | | 0.469053 | 0.2507 | | 0.9366 |
| | $\phi_{0.2}$ | | $\phi_{1.2}$ | $\phi_{0.2}$ | | $\phi_{1.2}$ |
| | 5.059136 | | 1.131816 | 0.9740 | | 0.0060 |

Source: Own calculations

Now, we will work with ARFIMA(0,d,1) model of time serie of temperatures measured at Prague and MSW model of time serie of temperatures measured at Prague. We proceed as follows: The historical data provides us with 13 values of *I*; we use these to obtain the historical sample mean, $Est(\mu_I^H)$. We now generate a very large number of index values using the fitted ARFIMA and MSW models and compute a close aproximation to $\mu_I^M$, the model population mean. Let us set $\Delta = \mu_I^M - Est(\mu_I^H)$. If $\Delta$ is large, then we should reject the model. To establish a rejection threshold, we need an estimate of the sampling fluctutations. This we obtain by dividing up the large model sample into 13 member sub-samples. Each sub-sample provides a sample mean, $Est(\mu_I^H)$, and we can use these to compute a value, $\Delta_{99}$, such that only 1 % of the sub-samples gives $\left| \mu_I^M - Est(\mu_I^M) \right| > \Delta_{99}$. Thus, we can reject the model with 99 % confidence if $\Delta > \Delta_{99}$. We can proceed analogously for $\sigma_I^2$.

For Prague, we consider a 5 mo HDD contract spannig November-March. We generate $2 \times 10^3$ index values to perform the test. Results are reported in Table 3 where
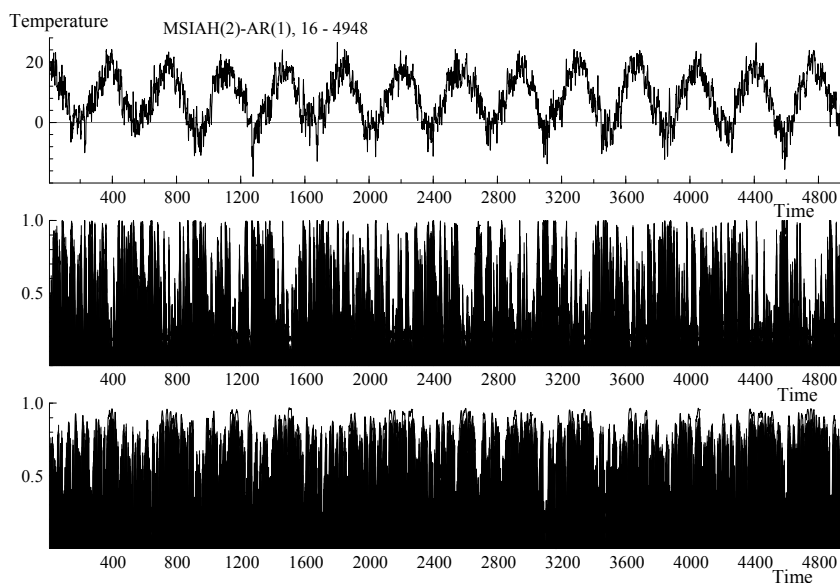
$\Delta$ indicates the difference between the historical and modeled value, as percentage of the latter. $\Delta_{99}$ indicates a 99% confidence level for $\Delta$.

**Figure 4: Est. time serie of temperatures measured at Prague and probability regimes of the MSW model**
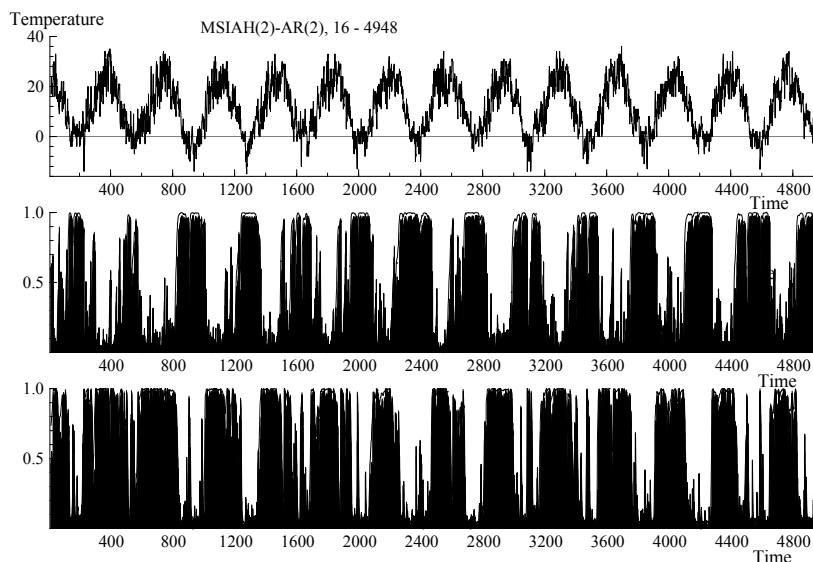


Source: Own calculations

**Figure 5: Est. time serie of temperatures measured at Brno and probability regimes of the MSW model**



Source: Own calculations

We see how we should expect on the basis of the results of tests (Portmanteau test, Asymptotic test, ARCH test) that the usage of the models to estimate the parameters of the probability distribution function (mean value, standart deviation) are not good. Only in the example second when we model the time serie of the temperatures measured at Prague by the MSW model we get better results. We see from this analyse that we have to target the volatility models to capture changeable variability. Linear and nonlinear models of time series of temperatures are not able to capture the minimum requirements set on the model which we define above.

**Table 3: Results of the simulation**

| | Historical | ARFIMA(0,d,1) | | | Markov-Switching model | | |
|---|---|---|---|---|---|---|---|
| | $Est\left(\mu_I^H\right)$ | $\mu_I^M$ | $\Delta$ | $\Delta_{99}$ | $\mu_I^M$ | $\Delta$ | $\Delta_{99}$ |
| **Prague** | 2410 | 2645 | 8.88 | 16 | 2534 | 4.89 | 9.6 |
| | $Est\left(\sigma_I^H\right)$ | $\sigma_I^M$ | $\Delta$ | $\Delta_{99}$ | $\sigma_I^M$ | $\Delta$ | $\Delta_{99}$ |
| **Prague** | 174 | 283 | 38.51 | 41 | 190 | 8.42 | 24.5 |

Source: Own calculations

# Conclusions

In the estimation of parameters $\mu_I$ and $\sigma_I$ of the probability distribution function *P(I)* we should reject linear and nonlinear models of time series of temperatures measured at Prague and at Brno and we should focus on volatility models which capture changeable variability in the sufficient measure which leads to better fulfilment of minimum requirements set on the model.

# References

[1]  ARLT, J.; ARLTOVÁ, M. (2007). *Ekonomické časové řady*. Praha : Grada, 2007.

[2]  CABALLERO, R. (2003). *Stochastic modeling of daily temperature time series for use in weather derivative pricing*. *Technical Report*. [on-line], Dublin, University College Dublin, c2003, [cit. 31. 3. 2007], <http://maths.ucd.ie/~rca/ publications/pdf/arfima.pdf>.

[3]  CABALLERO, R.; JEWSON, S.; BRIX, A. (2002). Long memory in surface air temperature: Detection, modeling, and application to weather derivative valuation. *Climate Research*, 2007, vol. 21, no. 1, pp. 1270ľ140.

# Deriváty na počasí

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## Abstrakt

Příspěvek se zabývá problematikou derivátů na počasí, které v současné době postupně nabývají na významu. Cílem této práce je definice derivátů na počasí a způsobu jejich ocenění Ukážeme také, že lineární a nelineární modely časových řad teplot měřených v Praze a Brně nám nedávají dobré výsledky v odhadu parametrů $\mu_I$ a $\sigma_I$ funkce pravděpodobnostního rozdělení *P(I)* indexu na počasí, který je při jejich oceňování podstatný.

**Klíčová slova:** deriváty na počasí; oceňování derivátů na počasí; časové řady.

# Weather Derivatives

## Abstract

The article deal with the problems of weather derivatives which take the value in sequence nowadays. The aim of this work is the definition of weather derivatives and the way how to price them. We show as well that linear and nonlinear models of time series of temperatures measured in Prague and in Brno have not good results in the estimation of parameters $\mu_I$ and $\sigma_I$ of the probability distribution function *P(I)* of the weather index which is essential in their pricing.

**Key words:** weather derivatives; pricing weather derivatives; time series.

**JEL classification:** C32.